Algorithmic Syntactic Causal Identification

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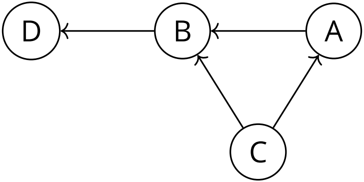
November 15, 2024

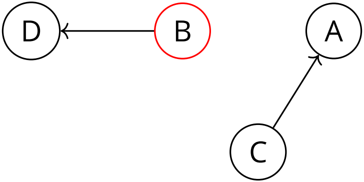
Basics of Causal inference

- Nodes represent random variables
- Directed arrows represent causal influence



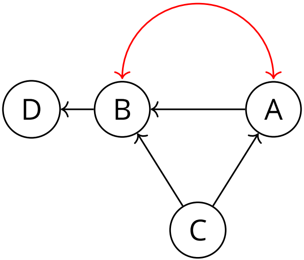
Basics of Causal inference

- Intervene by deleting parent edges
- Replace distribution with a constant distribution



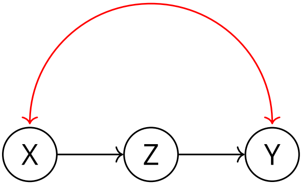
Basics of Causal inference

- In real life data is not always fully observed
- We can represent an unobserved confounde as a bidirected arc between two observed nodes



Causal identifiability

- Assess if we have enough information to answer a specific causal query
- A classic example is the *front-door* criterion

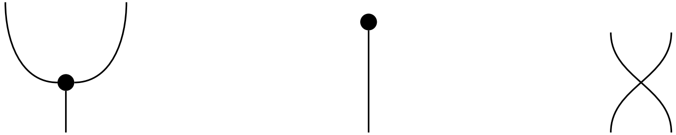


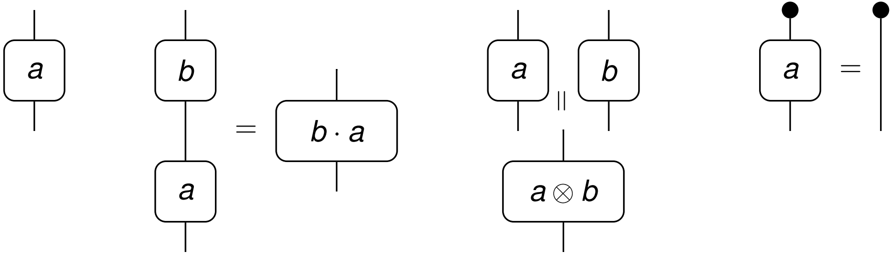
- E.g. Inferring whether or not smoking causes cancer in the lungs

Limitations of Causal inference

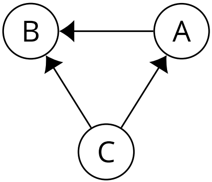
- How to represent Marginalization
- How to represent hidden/latent variables
- Conflation of syntax and semantics
- Causal queries outside of probability theory?

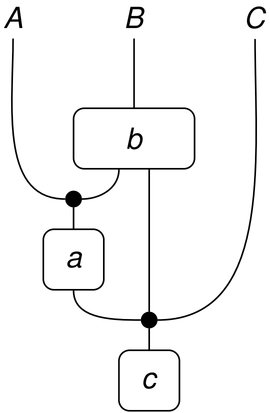
String diagrams





Building String Diagrams from DAGs





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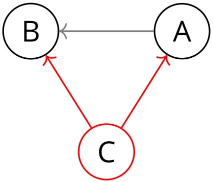

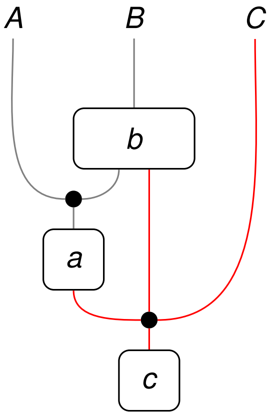

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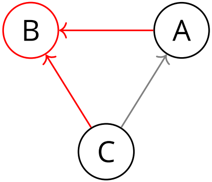
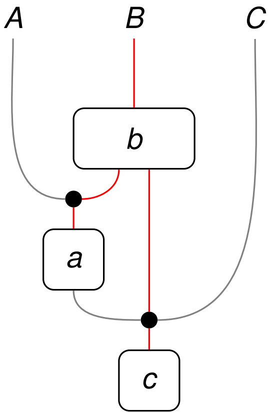
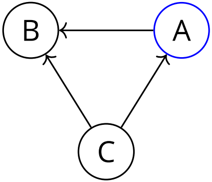
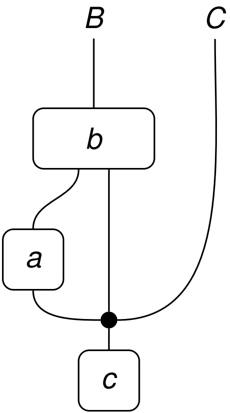


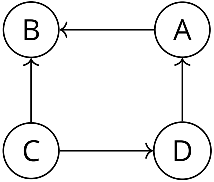


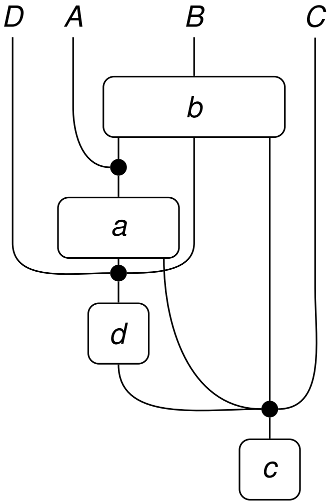
Representing unobserved variables



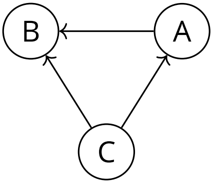


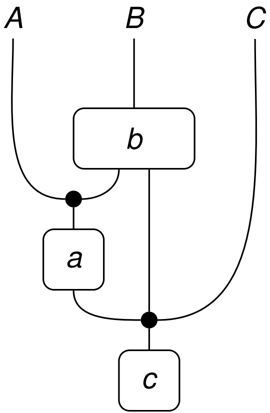
Factorized string diagrams





The fixing operation





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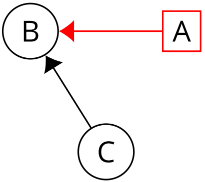
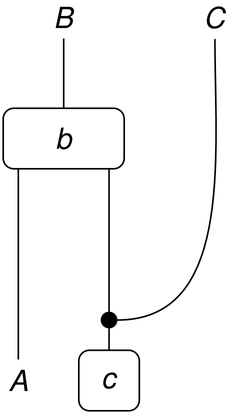
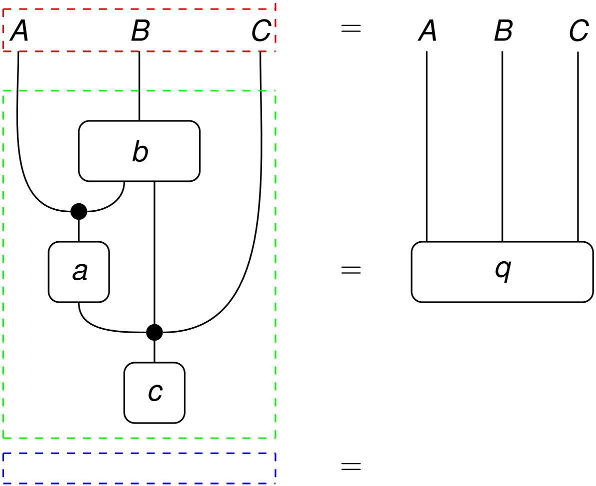
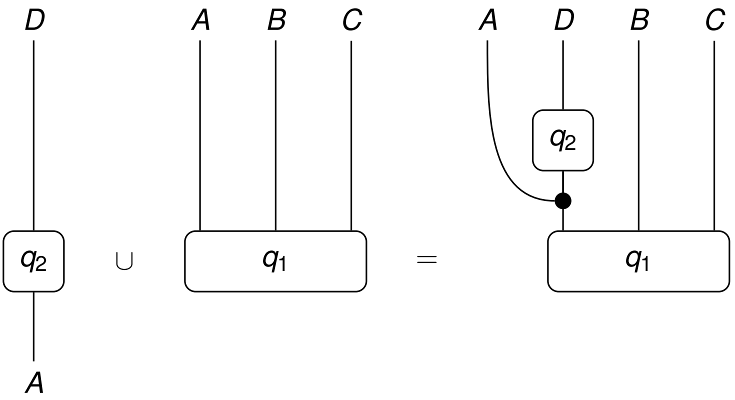




Exterior morphisms



Combining Operation



Casual Identifiability in a Syntactic setting

$$\Sigma^{\mathcal{G}}_{\mathbf{Y}|\mathrm{do}(\mathbf{A})} = \mathrm{Hide}_{\mathbf{Y}^{\star}\setminus\mathbf{Y}}\left(\bigcup_{\mathbf{D}'\in\mathbf{D}^{\star}} \mathrm{Simple}\left(\mathrm{Fixseq}_{\mathbf{V}^{\mathcal{G}}\setminus\mathbf{D}'}\left(\Sigma^{\mathcal{F}}\right)\right)\right)$$

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The factorized signature

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The valid sequence of applications of the fixing operation to be applied to the signature

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A purely algebraic formalism of the naturality of the delete operation

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Combing all the fixed and simplified signatures together, for each identified district

Casual Identifiability in a Syntactic setting

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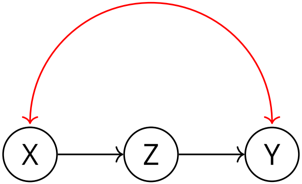
Applying the deletion operation on a set of morphisms in the final combined signature

Front-door Adjustment

- Start with front-door DAG and then identify valid fixing sequences



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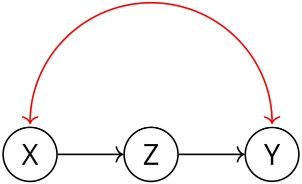

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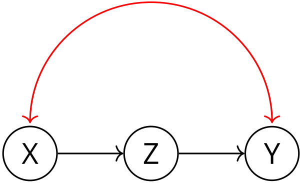

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- $\mathrm{Fixseq}_{\{X,Z\}} = \mathrm{Hide}_X \circ \mathrm{Fix}_Z$

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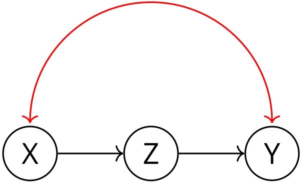

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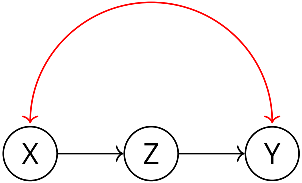
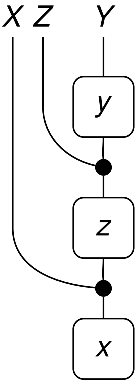
Front-door adjustment





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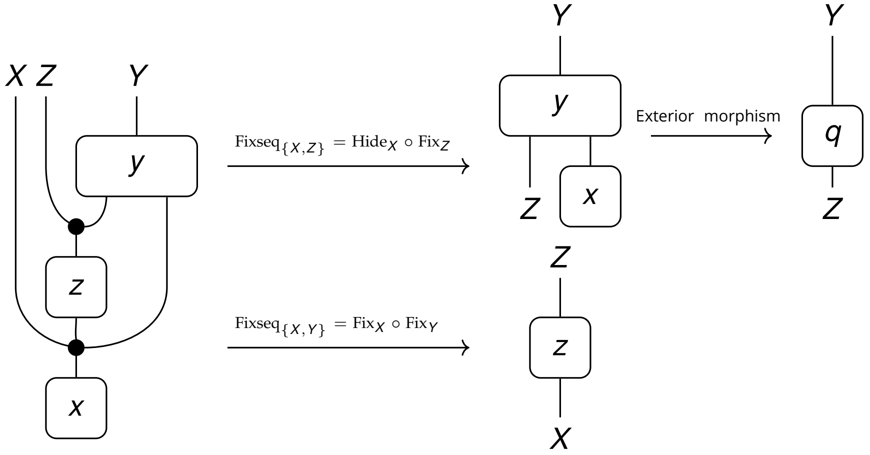
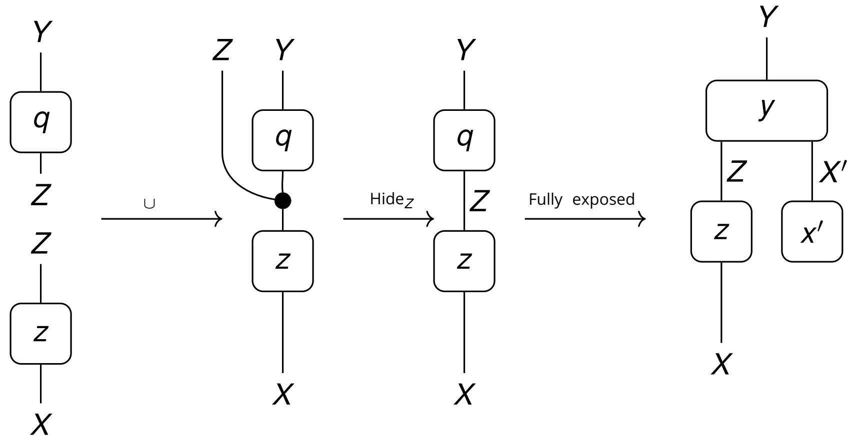
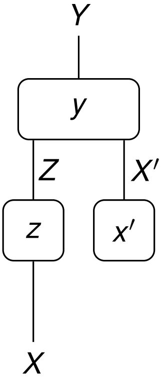


Front-door Adjusment



Interpretations



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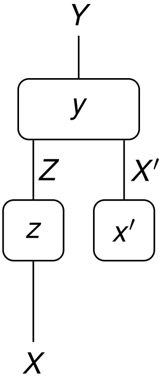

- $p\left(Y=y|\mathrm{do}\left(X=x\right)\right)=\sum_{z\in\Omega_Z}p\left(Z=z|X=x\right)\sum_{x'\in\Omega_X}p\left(Y=y|X'=x',Z=z\right)p\left(X'=x'\right)$

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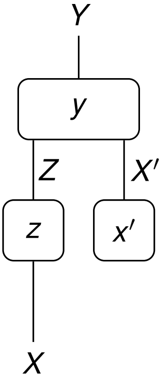

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- $f\left(y|do\left(x\right)\right)=\min_{z\in Z}\mathbf{f}\left(z|x\right)+\min_{x'\in X}\left[\mathbf{f}\left(y|x',z\right)+\mathbf{f}(x'|)\right].$

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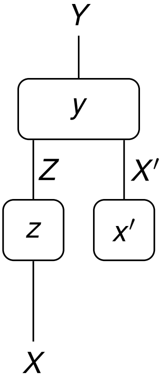

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- $f\left(y|do\left(x\right)\right)=\min_{z\in Z}\mathbf{f}\left(z|x\right)+\min_{x'\in X}\left[\mathbf{f}\left(y|x',z\right)+\mathbf{f}(x'|)\right].$
- $do\left(x\right)Ry \quad \text{iff} \quad \exists z,x'\in Z,X:(xR_3z)\wedge\Big(\left(\left(x',z\right)R_2y\right)\wedge\left(R_3x'\right)\Big)$

Conclusion

- We have introduced a purely syntactic form of causal identification
- This allows for causal identification in generic settings
- Interpretations can be implemented for non-probabilistic models