Hardware accelerated intelligent theorem proving

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GPU accelerated theorem proving

- GPUs really not made for theorem proving
- Are they still useful?
- Can we apply machine learning?

Videogames

- Problem: draw triangles fast
- Observation: we are doing the same operations many times
- Solution: stream multiprocessors

Stream multiprocessors

- Multiple ALUs with single instruction pointer
- Must ALL perform the same elementary instruction
  - Or run idle
- Sequentially 1 to 2 orders of magnitude slower than CPU
  - Ballpark estimate for similarly dated and priced hardware
- In parallel all SMs are 1 to 2 orders of magnitude faster than a CPU
  - Ballpark estimate for similarly dated and priced hardware

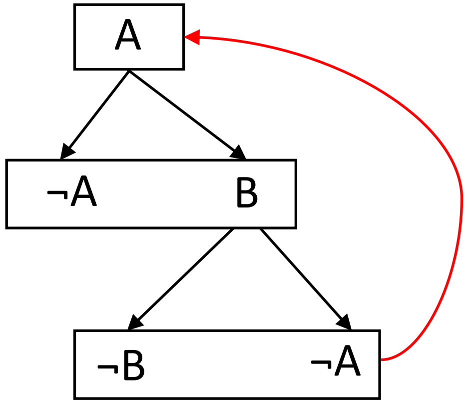
Thread divergence

- Goal: minimize thread divergence
- Easier for machine learning and physics simulations
- Hard for complex tasks (such as first order theorem proving)

Tableaux calculus

Unification

f(x, B)	f(A, y)	-> f(A, B)	
f(x, B, x)	f(A, y, y)	-> ☹	
f(x, g(x))	f(g(A), y)	-> f(g(A), g(g(A)))	
f(x, g(x))	f(y, y)	-> ☹	Occurs check



Tableaux calculus

- SAT → 1^{st} order
- Predicate logic has unique* most general unifiers
- Predicate logic is relatively complex
  - Predicate symbols, function symbols, different arities
- Lambda-free higher order logic also has unique* most general unifiers
- Lambda-free higher order logic is simple
  - Application (and of course constants and variables)
  - (hopefully) less thread divergence!

Tableaux calculus

- Substitutions are stored in small local dictionary
- Unification uses a custom stack (CUDA recursion bad)
- History is kept track of, proof steps are reversible

Proof exploration

- Iterative deepening is relatively hard and requires synchronization
- Randomly try steps instead
  - Far worse than iterative deepening, but easier to implement
- 1 million runs
  - Max 64 random steps for each run
  - If no proof is found: start over

Intermediate results

- M40 dataset
- Our method
  - 4783 out of 32444+ proofs found (some theorems were excluded due to size)
  - On average 1 second is spent on each theorem (using an RTX 3080)
  - 64 million inferences per second

Machine learning

- Tree neural network
  - Less advanced than graph neural network
- Neural Network evaluation is slow
- May detriment inferences per second
- Solution: evaluate neural network ahead of time

Machine learning

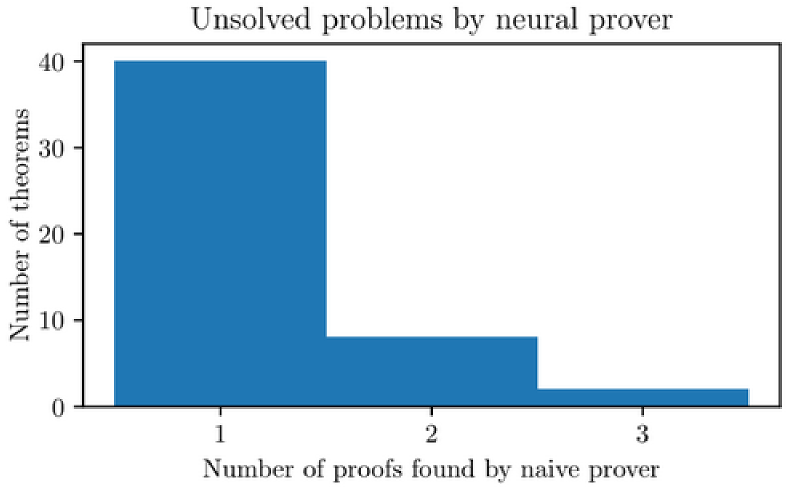
- Run prover
  - save proof steps of successful proofs
- Tally extension steps as pairs of literals, and normalize
- Train neural network on pairs of literals
  - Network has no state information
- Evaluate neural network on pairs of literals
- Run prover again, using evaluation as weights

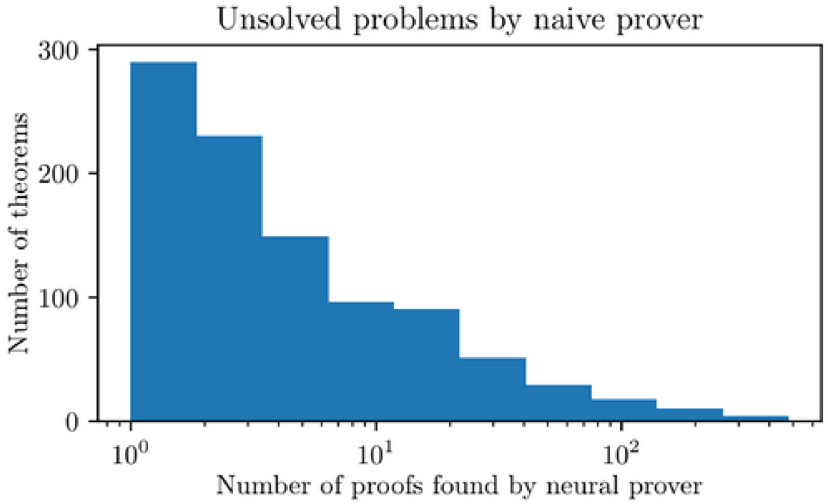
Machine learning

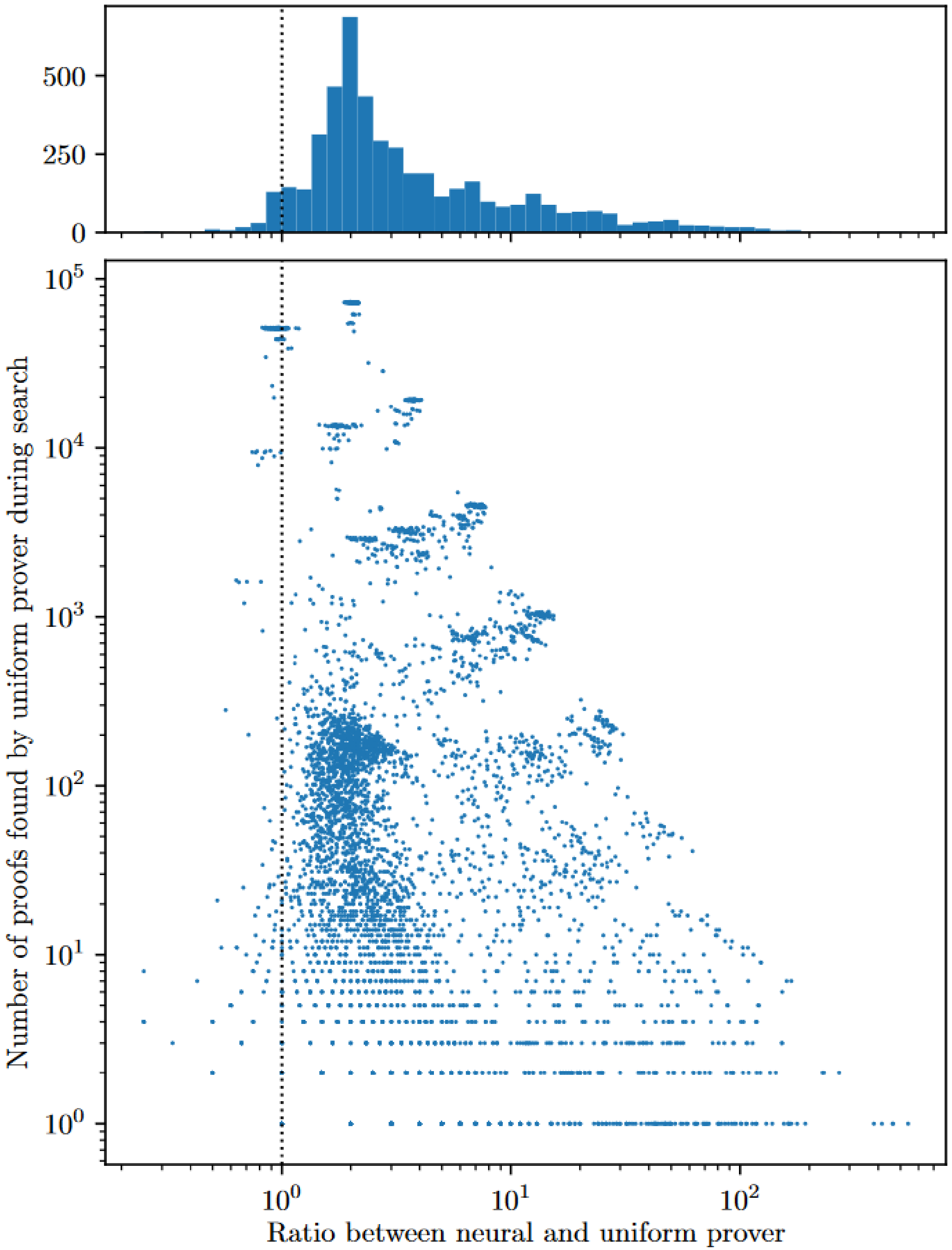
- Uniform prover: 4783 out of 32444 proofs found
- Neural prover: 5698 out of 32444 proofs found
- 965 newly proven problems
- 50 out of 4783 problems not proven by neural prover
- Any regressions?

(no) regressions

- Number of proofs found corresponds to Bernoulli distribution
- No regression hypothesis is plausible
- No improvement hypothesis is implausible







500
250
0
10^5
10^4
10^3
10^2
10^1
10^0
Number of proofs found by uniform prover during search
10^0
10^1
10^2
Ratio between neural and uniform prover

Future work

- Implement iterative deepening
  - Monte Carlo
- Better machine learning
  - Graph neural networks
  - Clause selection