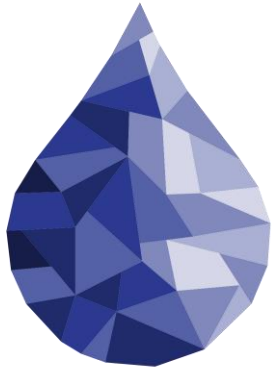

WATERPROOF

Educational Software for Learning How to Write Mathematical Proofs

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Motivation

Analysis 1

- **Goal:** teach students how to rigorously prove theorems from calculus
- Mathematical content

  metrics, sequences, series, limits, continuity
- Write valid proofs

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Proof Assistants in Education

Potential benefits

- Present **proving** as a game with clear rules and actions
- Immediate feedback
- Emphasize mechanical aspects of proving

Proof Assistants in Education

General trend

- PAT 2023 Thematic School
- Lean Together 2021: Panel on teaching with proof assistants
- Tobias Nipkow (2012): “*No More LSD Trip Proofs*”
- … instances going back to 70’s (Mizar)

Issues

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✓ Participants Lean panel
✓ Maria Knobelsdorf et al. (2017)
✗ Athina Thoma and Paola Iannone (2021)

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example of Coq proof

$$\forall \varepsilon \colon \mathbb{R}, \varepsilon > 0 \Longrightarrow \exists a \colon \mathbb{R}, (0 \leq a < 4) \wedge (4 - \varepsilon < a)$$

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$$\forall \varepsilon : \mathbb{R}, \varepsilon > 0 \Longrightarrow \exists a : \mathbb{R}, (0 \leq a < 4) \wedge (4 - \varepsilon < a)$$

```
intro ε. intro ε_gt_0.
assert (ε < 2 \/ 2 <= ε) as cases by lra.
destruct cases as [ε_lt_two | two_le_ε].
- (* Case ε < 2. *)
  set (a := 4 - ε/2); exists a.
  split.
  + assert (0 <= 4 - ε/2 < 4) as h1 by lra; exact h1.
  + assert (4 - ε < 4 - ε/2)  as h2 by lra; exact h2.
- (* Case ε ≥ 2. *)
  ...
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```
ε : R
ε_gt_0 :
  ε > 0
ε_lt_two :
  ε < 2
```

```
∃ a : R,
  ...
```

Waterproof

Design goals

- Suitable for use in educational environment
- Writing a proof in Waterproof should be **as close as possible to** writing a proof with pen and paper, both in terms of **style** and the **process** of constructing a proof

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Custom version of the Coq proof assistant

- Custom proof language
- Custom editor

Overview

- Introduction
- Demonstration
- Issue: working with subsets
- Conclusion

Custom proof language

Features

- Proof steps inspired by language in regular math proofs
- Implicit automation to verify statements
- Mandatory signposting
- Elaborate error messages
- Conventional mathematical notation
- Chains of (in)equalities

Custom editor

Features

- Mixed documents
- Continuous proof checking
- Designated input areas
- Hidden segments
- Limited automated bookkeeping
- Autcomplete for mathematical symbols and proof steps
- Separate panel for expanding definitions

Use in Education

Analysis 1

- First-year course
- ≈ 175 students (some retakes)
- Mathematics students (mandatory)

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- 8 weeks
- Lectures
- Instruction hours
  - 6 classes, ≈ 30 students per class
- Weekly homework exercises
  - Groups of 4
  - Within instruction classes

Use in Education

Waterproof

- Waterproof versions for selection of homework exercises
- Voluntary
  - Only 3/6 instructors can provide support for Waterproof
  - 1st and 2nd ‘line of defense’
- Not explicitly taught
  - Tutorial
  - Videos
  - Questions during instruction hours
- Automatic grading

Use in Education

Students' experience

- Small survey, conversations with students
- Some like using Waterproof, some don't
- Both stronger and weaker students
- High retention rate
  - Start: 25 homework groups
  - End: 19 homework groups

Use in Education

Results for handwritten proofs?

- Observations
- Improved readability
  - Students use controlled natural language formulations from Waterproof
- Improved proof structure

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Issue: subsets

Current approaches

- Sigma/record types ------> dealing with coercions ☹
- Classifying predicates -------> not directly usable with quantifiers

$$\forall x\colon \mathbb{R}, x > 0 \quad \text{instead of} \quad \forall x\colon \mathbb{R}_{>0}$$

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$$\forall x\colon \mathbb{R}, x > 0 \quad \text{instead of} \quad \forall x\colon \mathbb{R}_{>0}$$

Desired: behave like subsets in naive set theory

- given $P : X \to \mathrm{Prop}$, a type A such that
- If $x : A$ then $x : X$ and $P(x)$
- If $x : X$ and $P(x)$ then $x : A$

Conclusion

Waterproof

- Custom version of Coq for teaching how to write proofs
- Used in Analysis 1 course
- **Observation:** improved readability and structure

Conclusion

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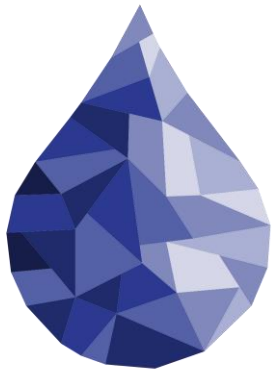
Future Work

- Proper evaluation
- Improve mathematical library



WATERPROOF

Educational Software for Learning How to Write Mathematical Proofs

- Editor https://github.com/impermeable/waterproof
- Proof language https://github.com/impermeable/coq-waterproof
- Exercises https://github.com/impermeable/waterproof-exercise-sheets

submitted article to post-proceedings

Workshop on Theorem proving components for Educational software (**ThEdu'23**)

References

- Lean Together 2021: Panel on teaching with proof assistants
  https://www.youtube.com/watch?v=mTLuON5eRZI&list=PLlF-CfQhukNnO8z3TcFcoKozif9gbl7Yt
- Tobias Nipkow (2012). *Teaching Semantics with a Proof Assistant: No More LSD Trip Proofs.*
- Krzysztof Retel and Anna Zalewska (2005). *Mizar as a Tool for Teaching Mathematics.*
- Maria Knobelsdorf, Christiane Frede, Sebastian Böhne, and Christoph Kreitz (2017).
  Theorem Provers as a Learning Tool in Theory of Computation.
- Athina Thoma and Paola Iannone (2021).
  Learning about Proof with the Theorem Prover LEAN: the Abundant Numbers Task.